Is Case-based Decision Theory Consistent with Empirical Patterns of Human Classification Learning?

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Basic question

- The Case-based Software Agent (CBSA) is an artificial problem solver.
- How does CBSA compare to human problem solvers?

What is the Case-based Software Agent?

Case-based

Case-based Decision Theory (Gilboa and Schmeidler 1995) is a *choice theory* for decision-making under uncertainty.

Software Agent

A software agent is “an encapsulated piece of software that includes data together with behavioral methods that act on these data.(Tesfatsion 2006)”

Human Problem Solvers

- Shepard, Hovland, and Jenkins (1961) study how people learn how to classify objects.
- Test subjects learn the correct classifications through immediate feedback on repeated sorts.
- They find some classifications are harder for people to learn than others.
- They also learned how fast people learn classifications.

How to answer the basic question.

1. Define the classification problem for CBSA and test his behavior.
2. Empirically compare CBSA's choice behavior with human choice behavior.

What is new here?

- How CBDT compares to human test subjects on classification learning has never been done before.

  But that's not it.

- Empirical work in EDT is either
  1. Thought experiments
  2. (in the last ten years) Laboratory experiments about simple blackboard results
- Never before has there been a way to generate large amounts of nuanced choice data directly implied by an (economic) decision theory, that can be brought to *existing data* outside of EDT

Rest of this talk

1. What is Economic Decision Theory? What's Case-based Decision Theory?
2. What is Agent-based Computational Economics?
3. Based on points 1,2: how does CBSA work?
4. The Classification Learning problem: Humans and CBSA
5. Empirical comparison of CBSA and Human data
6. Going back to the big picture.

What is (Economic) Decision Theory?

- Definition: A Decision Theory is a mathematical model of PREFERENCES and INFORMATION that will predict CHOICE when outcomes are UNCERTAIN.

- The first Decision Theory: von Neumann and Morgenstern's (1944) Expected Utility Theory (EUT).
  1. PREFERENCES are represented by a utility function over outcomes;
  2. INFORMATION is represented by a set of possible states of the world;
  3. UNCERTAINTY is represented by a probability distribution over states;
  4. And an agent CHOOSES an action which maximizes the probability-weighted sum of utilities.

Case-based Decision Theory (G&S 1995)

- Agent judges ‘how similar is the problem I face today to experiences in memory?’
- What choices were taken? What were the results?
- Agent constructs an expectation of outcome for each action based on that similarity.
- Agent chooses the act with highest similarity-weighted result.

An example: Finding a Nanny

Your job is to find nannies for families. Family 0 enters your office. There are two possible nannies: A and B. A has references from families 1 and 2, and B has references from families 2 and 3.

- Available actions $\{A, B\}$ (Choose an available nanny)
- Cases in memory: the set of references $\{A1, A2, B2, B3\}$
- Set of problems $=$ Set of families $0, 1, 2, 3$
- Results: How happy was the family in each case $\{A1, A2, B2, B3\}$?
- Similarity: How similar is family 0 to family 1? To family 2? To family 3?

Formally: Case-based Decision Theory

- Set of problems $p \in \mathcal{P}$
- $a \in \mathcal{A}$ is an action chosen in response to a problem
- $r \in \mathcal{R}$ is a outcome
- A case is a vector $\{p, a, r\}$, Memory is a set of cases

- A similarity function $s(p, q)$, $p, q \in \mathcal{P}$
- A memory $M(a)$, which is a set of which act a was taken
- Act a, under problem p, is evaluated according to:

$$CBU(a) = \sum_{\{q,a,r\} \in M(a)} s(p, q)u(r)$$

Agent-based Computational Economics

Agent-based Computational Economics (ACE) seeks to simulate economic models through explicitly defining software agents and allowing them to interact.

See also: Agent-based Modeling, Complex Systems, Complex Adaptive Systems.

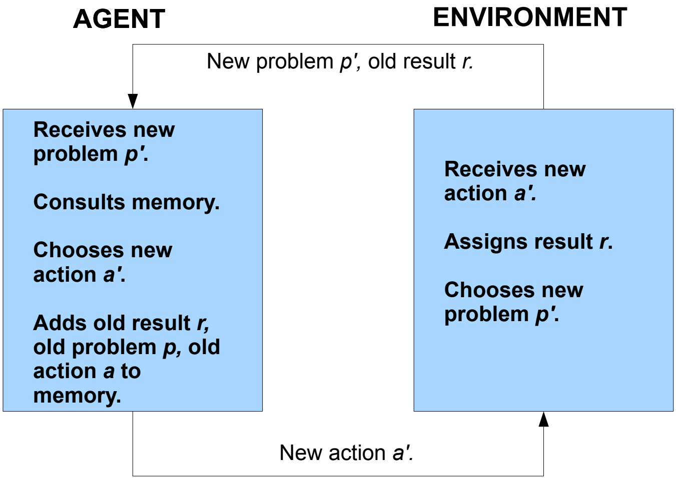
An example: Hill climbing

Try it:

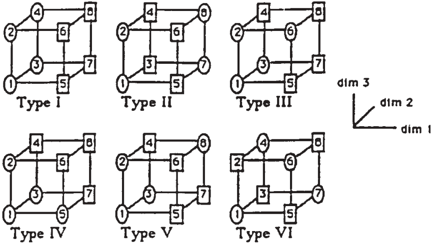
1. Open http://roboeconomic.us/cbsa. You can control the agent from there.
2. Choose FindWell from the Problem_Type pull-down menu.
3. Click 'Setup' once and 'Step' repeatedly, to watch him hillclimb. ('Go' repeats 'Step.')
4. Try increasing the number of agents (Nagents), then repeat step 3.

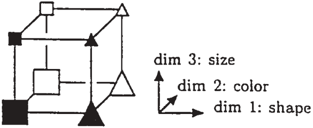
- CBSA gets higher utility at each step, the closer he ends up to the well.
- Each step is a "problem," and the agent remembers his entire path.
- CBSA judges similarity based on Euclidian distance.

An Agent's-Eye View of the World



The Classification Learning Problem





The problem: sorting three-digit binary strings into two categories. *(Image from Nosofsky et al. 94.)*

CBSA and the Classification Learning Problem

Try it:

1. Open http://roboeconomic.us/cbsa. You can control the agent from there.
2. Choose ClassificationProblem from the Problem_Type pull-down menu.
3. Click ‘Setup’ once and ‘Step’ repeatedly, to watch him classify objects. (‘Go’ repeats ‘Step.’)
4. Try changing Classification_Type, then repeat step 3. This changes from problem Type I (1) to Type VI (6).

Notice:

- The agent makes fewer mistakes on Type I than Type VI.

A Comparison of # Errors by Problem Type (CBSA)

	Ttl	I	II	III	IV	V	VI
MEAN	6.42	3.69	7.65	6.50	5.51	7.13	8.04
STD	1.82	1.05	1.14	1.24	1.40	1.00	0.20
MIN	2	2	4	4	2	6	8
MAX	10	6	8	8	7	10	9
N	12K	2K	2K	2K	2K	2K	2K

(Note: All means pass pairwise T-tests of difference at the 0.01% *level.)*

- We find the CBSA finds the problems to be of this order of difficulty:
  Type I $<$ Type IV $<$ Type III $<$ Type V $<$ Type II $<$ Type VI.
- The classic human empirical result by Shepard, Hovland, and Jenkins (1961) is:[2]
  I < II < III, IV, V < VI
- Nosofsky, Gluck, Palmeri, McKinley, and Glauthier (1994) find in a related human study that:
  I < IV < III < V < II < VI
  Which matches CBSA. This result relies on item confusability.

[2] In recent work, Kurtz finds I < II, III, IV, V < VI

Speed

Humans make mistakes well into their tenth “block” of sixteen trials.

CBSA never makes more than ten mistakes, and never after the first “block.”

Conclusions

Narrow Conclusions:

- CBSA shows that Case-based Decision Theory empirically matches the problem type order of human classification with item confusability, but not without it
- CBSA also shows that Case-based Decision Theory implies unrealistically quick learning

But, more broadly:

- CBSA is a new tool for Decision Theorists to empirically evaluate choice theories

The Future

- How does item confusability relate to Gilboa and Schmeidler's own understanding of CBDT? Is there insight there?
- Attentional weights and Empirical Similarity: can an evoloving similarity function bring the CBSA results closer to the classic result?
- CBSA and learning speed: is there an *interesting* way to modify CBDT to bring CBSA closer to empirical reality?
- CBSA in other economic agent-based models: useful where a more sophisticated agent is required?
- Other decision theories to implement and test?

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(Appendix slides to follow)

The Classification Problem in EDT-speak

- $\mathcal{P}$ is the complete set of three-digit binary strings
- Let $C = \{c_1, c_2\}$ be the set of categories
- Nature determines the true relationship between strings and categories with the function $F : B \to C$.
- The agent observes a particular string p and must announce a guess $g \in C$.
  - If $g = F_t(b)$, i.e. if the guess is correct, the agent wins (receives a utility payoff of $r_0 = 0$)
  - Otherwise, the agent loses (payoff of -1)
- Run CBSA on this problem.

Representation Theorems

The Origins of Decision Theory

- von Neumann and Morgenstern (1944):
    - If: Choice behavior satisfies these axioms w/r/t a risky choice w/ known probabilities
    - Then: $u(x)$ s.t. the agent is an Expected Utility Maximizer.
- Savage (1954):
    - If: Choice behavior satisfies additional axioms, but unknown probabilities
    - Then: $u(x)$ and probabilities, s.t. Subjective Expected Utility Maximizer

The Split in Decision Theory

- Philosophy
  - Causality and State Spaces
- Artificial Intelligence
  - A.I. *per se*: solving complicated problems
  - Cognitive Science: solving problems as humans do
- Economics
  - 'Behavioral Economics': formal model adaptations from psychology

Risk, Uncertainty, and “Greater Ignorance”

- Risk: a set of states with known probabilities v.
- Uncertainty: a set of states with unknown probabilities (Knight 1921)
- A “new kind of ignorance”(Gilboa and Schmeidler 1995): when set of states unknown.

Appropriate for: decisions in which situation is never repeated, state space difficult to imagine, or problem not well-understood in principle.

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